Math 122 **Name:** ______________________________

Spring 2024

Exam 1 Practice

1/29/2024

Time Limit: 75 Minutes Signature: ______________________________

This exam has 7 questions, for a total of 70 points and 0 bonus points. Unless otherwise specified, there is no form of technology allowed. Further, final solutions must be written in the prescribed boxes, and all work must be shown. There is paper provided in the front of the class for scratch work. Any numerical values given for a final answer must be precise.

Grade Table (for teacher use only)

Question	Points	Bonus Points	Score
1	10	0	
2	10	0	
3	10	0	
4	10	0	
5	10	0	
6	10	0	
7	10	0	
Total:	70	0	

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34 to 43 at end of Chp 2

1. (10 points) Show the following: If $f(x) = \frac{1}{x}$, then $f'(x) = -\frac{1}{x^2}$.

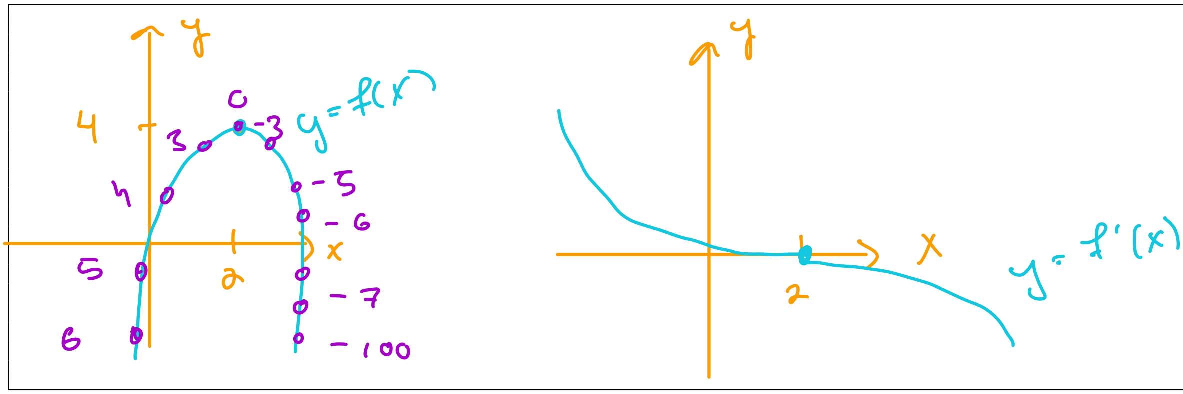
$$f'(x)\left(\frac{dy}{dx}\right) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{\left(\frac{1}{x+h}-\frac{1}{x}\right)}{h}$$

$$= \lim_{h\to 0}\frac{\left(\frac{x-(x+h)}{x(x+h)}\right)}{h} = \lim_{h\to 0}\frac{\left(\frac{-h}{x(x+h)}\right)}{h} = \lim_{h\to 0}\frac{-\cancel{h}}{x(x+h)}\cdot\frac{1}{\cancel{h}}$$

$$= \lim_{h\to 0} -\frac{1}{x(x+h)} = -\frac{1}{x^2}$$

$$\left[\frac{\left(\frac{a}{b}\right)}{\left(\frac{c}{d}\right)} = \frac{ad}{bc}\right] \quad (*)$$

2. (10 points) Sketch a graph of the derivative of the function $f(x) = -(x-2)^2 + 4$. Hint: Graph $f(x)$ from $y = x^2$ using transformations.



3. (10 points) Compute the compositions $f\circ g, g\circ f, f\circ f$ where $f(x) = \frac{1}{x}$ and $g(x) = 3x+4$.

$$f(g(x)) = f(3x+4) = \frac{1}{3x+4}$$

$$g(f(x)) = g\left(\frac{1}{x}\right) = 3\cdot\left(\frac{1}{x}\right)+4 = \frac{3}{x}+4$$

$$f(f(x)) = f\left(\frac{1}{x}\right) = \frac{1}{\left(\frac{1}{x}\right)} = \frac{1}{1}\cdot\frac{x}{1} = x \quad \text{(See (*))}$$

$\left\lceil \ln\left(\frac{a}{b}\right) = \ln(a) - \ln(b) \right\rfloor$

4. (10 points) Solve for t where $5e^{3t} = 8e^{3t}$. $\lceil \ln(ab) = \ln(a) + \ln(b) \rfloor$

$5e^{3t} = 8e^{3t} \underset{(\ln)}{\Longrightarrow} \ln(5e^{3t}) = \ln(8e^{3t})$

$\Longrightarrow \ln(5) + \ln(e^{3t}) = \ln(8) + \ln(e^{3t})$

$\Longrightarrow \ln(5) + 3t\,\cancel{\ln(e)} = \ln(8) + 3t\,\cancel{\ln(e)}$

$\Longrightarrow \ln(5) + 3t = \ln(8) + 3t$

$\Longrightarrow \ln(5) = \ln(8)$, impossible

$\Longrightarrow$ no solutions

$\lceil\, 5e^{3t} = 8e^{3t} \Rightarrow \frac{5e^{3t}}{e^{3t}} = \frac{8e^{3t}}{e^{3t}} \Rightarrow 5 \neq 8 \,\rfloor$

5. (10 points) Find the average rate of change of $f(x) = 3x^2 + 4$ between $x = -2$ and $x = 1$.

$\lceil\, \frac{\Delta y}{\Delta x}$ of f on (a,b) is $\frac{f(b) - f(a)}{b-a} \,\rfloor$

$\frac{\Delta y}{\Delta x} = \frac{f(1) - f(-2)}{1-(-2)} = \frac{3\cdot 1^2 + 4 - (3\cdot(-2)^2 + 4)}{3}$

$= \frac{3 + \cancel{4} - 12 - \cancel{4}}{3} = \frac{-9}{3} = -3$

6. (10 points) Find an equation for the line passing through $(4, 5)$ and $(2, -1)$.

$m = \frac{\Delta y}{\Delta x} = \frac{-1-5}{2-4} = \frac{-6}{-2} = 3$

$y - 5 = 3(x-4) \Rightarrow y = 3x - 12 + 5 = 3x - 7$

$\lceil\, y = mx + b \Rightarrow 5 = 3x + b \Rightarrow b = \cdots \,\rfloor$

7. (10 points) Determine the slope and y-intercept of the line given by $7y + 12x - 2 = 0$.

$7y + 12x - 2 = 0 \implies 7y = 2 - 12x$

$\implies y = \frac{2}{7} - \frac{12}{7}x \quad (= mx + b)$

$m = -\frac{12}{7}$

$y\text{-int} = \frac{2}{7}$